Section 5.2 Polynomials, Sums, and Differences

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Learning Objectives:

- Give the degree of a polynomial
- Add and subract polynomials
- evaluate a polynomial for a given value of its variable

Definition

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The numerical factor of a term is a **coefficient**.

For example, the aforementioned terms have coefficients $100, 3, 5$, and $\frac{2}{3}$.

Definition

A **constant** is a single number, such as 8 or 9.

Definition

A **monomial** in one variable is the *product* of a constant (a number) and a variable raised to a whole number $(0, 1, 2, \ldots)$ power. A monomial in one variable has the form

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For instance,

$$3, \quad 5x, \quad 7x^4, \text{ and } \quad 9x^{200}$$

are all examples of monomials

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The degree of a nonzero constant is zero. Because $0 = 0x = 0x^2 = 0x^3 = \ldots$, we cannot assign a degree to the number 0. Therefore, we say 0 has no degree.

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- $4x^{-3}$ is not a monomial because the exponent of the variable, x, is -3 and -3 is not a whole number.
- $2x^{1/3}$ is not a monomial because the exponent of the variable is $1/3$, and $1/3$ is not a whole number.

Definition

A **polynomial** is a monomial or a sum of monomials.

Polynomials are sums of monomials.

11	monomial
$3x^4$	monomial
$2x^2 + 1$	called a binomial because it has two terms
$5x^3 + x - 1$	called trinomial because it has three terms
$x^{1/2} + 5$	is not a polynomial
$\sqrt[5]{x+5}$	is not a polynomial
$\dfrac{1}{x-1}$	is not a polynomial

Definition

Like terms are terms that contain the same variable(s) raised to the same power(s). Like terms can be combined or collected together by writing them as a single term whose coefficient is the sum of the coefficients of the terms being combined.

Example Identify the like terms of the following polynomial:

$$4x^3 + 5x - 7x^2 + 2x^3 + x^2$$

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Solution:

like terms: $4x^3$ and $2x^3$ same variable and exponent

like terms: $-7x^2$ and x^2 same variable and exponent

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Solution:

$4x^3$ and $2x^3$ can be combined into $6x^3$ using the distributive property:

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Example Identify the like terms of the following polynomial:

$$4x^3 + 5x - 7x^2 + 2x^3 + x^2$$

Solution:

$-7x^2$ and x^2 can be combined into $-6x^2$ using the distributive property:

$$-7x^2 + x^2 = -7x^2 + 1x^2 = (-7 + 1) \cdot x^2$$

Example: Subtract $(x^2 - 5x) - (3x^2 - 4x - 1)$

$$(x^2 - 5x) - (3x^2 - 4x - 1) =$$

$$= (x^2 - 5x) - 1(3x^2 - 4x - 1) \qquad \text{since } -a = (-1) \cdot a$$

$$= (x^2 - 5x) + (-1)(3x^2 - 4x - 1) \qquad \text{since } a - b = a + (-b)$$

$$= (x^2 - 5x) - 3x^2 + 4x + 1 \qquad \text{distr. prop}$$

$$= x^2 - 5x - 3x^2 + 4x + 1 \qquad \text{assoc. prop}$$

$$= (x^2 - 3x^2) + (-5x + 4x) + 1 \qquad \text{comm. and assoc. props}$$

$$= \boxed{-2x^2 - x + 1} \qquad \text{addn closure prop}$$